51. IWK

Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

PROCEEDINGS 11-15 September 2006

FACULTY OF ELECTRICAL ENGINEERING AND INFORMATION SCIENCE

INFORMATION TECHNOLOGY AND ELECTRICAL ENGINEERING - DEVICES AND SYSTEMS, MATERIALS AND TECHNOLOGIES FOR THE FUTURE

TECHNISCHE UNIVERSITÄT
ILMENAU

Impressum

Herausgeber: Der Rektor der Technischen Universität Ilmenau
Univ.-Prof. Dr. rer. nat. habil. Peter Scharff

Redaktion: Referat Marketing und Studentische Angelegenheiten
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Redaktionsschluss: 07. Juli 2006

Technische Realisierung (CD-Rom-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Marco Albrecht
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag: isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilrnenau

ISBN (Druckausgabe): 3-938843-15-2
ISBN (CD-Rom-Ausgabe): 3-938843-16-0

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=12391

51st Internationales Wissenschaftliches Kolloquium
Technische Universität Ilmenau
September 11 – 15, 2006

J. Naumenko, V. Astakhov

Computing of Steady and Pseudo-Steady Magnetic Fields in Presence of Ideal Conducting Surfaces

APLIED ELECTROMAGNETICS AND CIRCUIT THEORY

A stationary or quasi-stationary magnetic field of induction $\overline{B}$ is examined in presence of finite multi-connected surface S with boundaries, on which condition of the ideal conductivity is satisfied:

$$\overline{B}\,\overline{n} = 0 \text{ on } S, \tag{1}$$

where $\overline{n}$ is a unit vector of a normal to S.

There is potential $\overline{A}$: $rot\overline{A} = \overline{B}$, $div\overline{A} = 0$. The condition (1) looks as $rot_n\overline{A} = 0$ on S, that is $\overline{A} = \overline{C}$ on S. $\overline{C}$ - some field, $rot_n\overline{C} = 0$.

We suppose the surface an it's boundaries satisfie to conditions of Lipschitz and magnetic field of the reaction has finite energy. Also we suppose the energy of nonperturbed magnetic field is bounded in some finite domain G i.e. $\iiint\limits_G \left|\overline{B^0}\right|^2 dG < \infty$. Borders of G satisfy to conditions of Lipschitz; $S \subset G$.

We consider the space $L_2(S)$ of quadratically integrable complex-valued vectorial functions $\left\{\overline{a_k}\right\}$ on S. Vectorial space $L_2(S)$ can be decomposed on S to a sum of orthogonal subspaces : $L_2(S) = L_2^{(\Pi)}(S) \oplus L_2^{(\Gamma)}(S) \oplus L_2^{(C)}(S)$ where $L_2^{(\Pi)}(S)$ consists of generalized by Weyl potential fields, $L_2^{(C)}(S)$ generalized solenoidal fields, and $L_2^{(\Gamma)}$ generalized harmonic fields [1,2]. We use space $\mathfrak{L} = L_2^{(\Gamma)}(S) \oplus L_2^{(C)}(S)$.

We designate $\mathrm{P}^{(\mathfrak{L})}$ the operator of orthogonal projection of $L_2(S)$ to $\mathfrak{L}$. Surface currents density $\overline{\sigma}$ can be described by integral equation of the first kind [3]:

$$\mathrm{T}\overline{\sigma} = \overline{f}, \tag{2}$$

where

$$\mathrm{T} = \mathrm{P}^{(\mathfrak{L})}\Gamma, \quad \Gamma\overline{a}(M) = \frac{1}{4\pi}\iint\limits_S \frac{\overline{a}(N)}{r_{NM}} dS_N ,$$

$$\overline{f}=-\frac{1}{\mu}\mathrm{P}^{(\mathfrak{L})}\left(\overline{A^{0}}-\overline{A^{*}}\right),\ \overline{A^{*}}(M)=\sum_{j=1}^{m}\frac{\Phi_j}{4\pi}\mathrm{rot}\oint_{l_j}\frac{\overline{dl_N}}{r_{NM}},\ M\in S.$$

where $\overline{A^{0}}(M)$ is a potential of the nonperturbed magnetic field of external (given) sources; μ - a magnetic permeability; $\overline{\sigma}$ - a surface currents density; r_{NM} - distance between points N, M, Φ_i - given flow of magnetic field through the hole i in surface S, l_i - some closed curve passing throw the hole i.

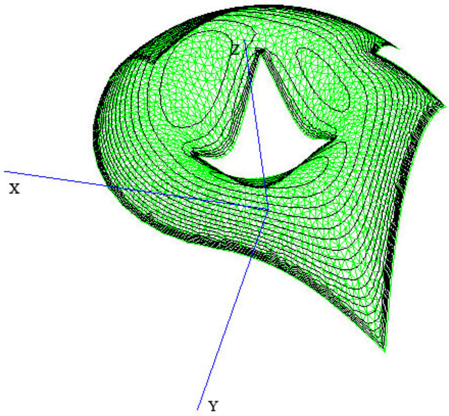

Fig. 1

It is possible to prove that the solution of the equation (2) does not only exist and is unique in power space $\mathfrak{L}_T$ of the operator T, but also $\mu\left\|\overline{\sigma}\right\|^2_{\mathfrak{L}_T}\leq\frac{1}{\mu}\iiint_{\Omega}\left|\overline{B^0}\right|^2 d\Omega$. Last inequality means in particular that the error of the magnetic field energy of reaction does not exceed a similar error of the external sources field energy. The solution of the equation (2) is numerically stable in this sense.

The simple form of kernel of integral equation (2) opens broad possibilities to optimization of numerical algorithms for their solving. A software package was built on the basis of the described theory. The example of the numerical method usage for complex-form surface with hole is showed in fig. 1. The magnetic field of given sources is homogeneous.

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